Velocity of Cavitation Bubbles in Uniform Flowfield High and Low Reynolds Number

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Abstract

A GASEOUS bubble caused by the tip cavitation of a propeller blade operating in a uniform flowfield environment experiences a drag force in relation to its angular velocity and a thrust force in the axial direction. In this paper, a mathematical model is presented that studies propagation of the gaseous bubble at low and high Reynolds number cases. The temporal and the spatial profile velocity variations are presented. The time constants involved in both regimes are obtained. The velocity variations are shown to be exponential for the low Reynolds number case and algebraic for the high Reynolds number case.

Contents

When a propeller operates in an underwater environment at high revolutions, cavitation will occur. Three different types of cavitation can occur: 1) hub cavitation, 2) blade cavitation, and 3) tip cavitation. In this paper, tip cavitation will be considered. Tip cavitation produces tip vortices of entrained gases that propagate downstream in a helical motion. The rate at which the cavitation gases propagate downstream is a function of propeller geometry, flowfield, and the geometry of the cavitation gases. The three different types of cavitation that can exist are 1) individual spherical bubbles of equal size, 2) individual bubbles of different sizes, and 3) train of gas shaped in cylindrical form. In this paper, it will be assumed that the cavitation bubbles are spherical and the shape does not change as a function of time. The motion of air bubbles in a fluid has been investigated by a variety of authors with numerous articles being presented by Lauterborn.[1]

Individual gaseous bubbles will be assumed to be moving in a flowfield, and interactions between successive bubbles will be neglected. The bubble is assumed to move with velocity V, whereas the flowfield has a velocity U. The effect of the added mass and the mass of the displaced fluid have to be considered in the general governing equation.

The governing equation of motion is the conservation of momentum equation as given by Newman[2]

$$m_g \frac{d\bar{V}}{dt} = -m_a \frac{d\bar{V}}{dt} + (m_g + m_d)\frac{\partial \bar{U}}{\partial t} + (m_g + m_d) \times (\bar{V} - \bar{U})\frac{\partial \bar{V}}{\partial x} - \bar{F} \tag{1}$$

where the terms m_g, m_a, and m_d are the mass of the gaseous bubble, the added mass for the bubble, and the mass of the displaced fluid, respectively. The F term is the drag (or thrust) acting on the gaseous bubble due to the fluid viscosity. For a spherical bubble, the drag (or thrust) term F is given by

$$\bar{F} = \tfrac{1}{2}\rho C_D R^2 \, |\bar{V} - \bar{U}| \, (\bar{V} - \bar{U}) \tag{2}$$

The drag coefficient C_D is that of a sphere and is a function of the relative velocity $(V - U)$. Lauterborn[1] has shown that the low Reynolds number drag coefficient of a gaseous bubble moving in a liquid is twice the drag of an equivalent sphere with rigid walls. The drag coefficient C_D is thus chosen as given by White,[3]

$$C_D = 0.4 + \frac{6}{1 + \sqrt{Re}} + \frac{48}{Re} \tag{3}$$

where Re is the relative Reynolds number.

At very low Reynolds numbers, the Stokes approximation dominates the drag coefficient term, whereas at the high Reynolds numbers, the constant term 0.4 will dominate. In most cases of interest, the gas density can be neglected with respect to the fluid density. The nonlinear term is neglected and the flowfield is assumed to be steady state with a constant velocity U_o. Under the aforementioned assumptions, the two coupled differential equations representing the axial and the circumferential bubble velocity can be decoupled. The result is two ordinary differential equations for the bubble axial velocity component V and the bubble angular velocity ω as a function of the time constant parameter τ_o.

$$\tau_o \frac{dV}{dt} + V = U_o \tag{4}$$

$$\tau_o \frac{d\omega}{dt} + \omega = 0 \tag{5}$$

The solutions for the two velocity components are then

$$\frac{V}{U_o} = 1 - \exp(-t/\tau_o), \qquad \frac{\omega}{\omega_o} = \exp(-t/\tau_o) \tag{6}$$

The only parameter of the solution is the time constant τ_o, which is equivalent to the time required for the initial conditions to decay within e^{-1} of the final conditions.

Figure 1 shows the solution for V and ω as a function of the dimensionless time t/τ_o. To evaluate the bubble path, integration of the velocity terms will give the axial and angular displacements, $x(t)$ and $0(t)$. Since buoyancy and lateral dispersion effect are neglected, the bubbles will travel in a helical motion.

At high Reynolds numbers, the equations are also amenable to a closed-form solution. For this condition, the drag coefficient of a spherical drop is a constant and is equal to 0.4. The two governing momentum equations cannot now be decoupled and are given by

$$\tau_1 U_o \frac{dV}{dt} + (V - U)[(V - U)^2 + (\omega R)^2]^{1/2} = 0 \tag{7}$$

$$\tau_1 U U_o \frac{d}{dt}(R\omega) + \omega R\,[(V\text{-}U)^2 + (\omega R)^2]^{1/2} = 0 \tag{8}$$

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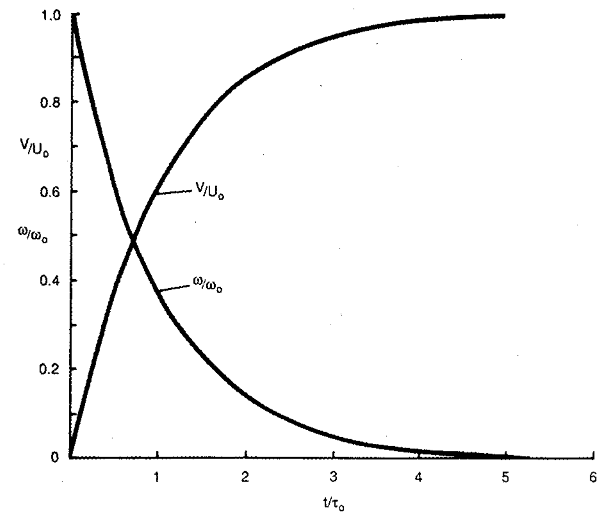


Fig. 1 Temporal variation of axial and angular velocity components: low Reynolds numbers.

In order to obtain simple closed-form solutions, two separate conditions are considered: 1) low swirl rates $\omega R \ll |U\text{-}V|$, and 2) high swirl rates $\omega R \gg |U\text{-}V|$. For the low swirl rates, the solutions are given by

$$\frac{V}{U_o}=\frac{t/\tau_1}{1+(t/\tau_1)}, \qquad \frac{\omega}{\omega_o}=\frac{1}{1+(t/\tau_2)} \tag{9}$$

Integrating these two equations with respect to time will then give the axial and circumferential location as a function of time.

For large swirl rates, closed-form solutions can also be obtained and are given by

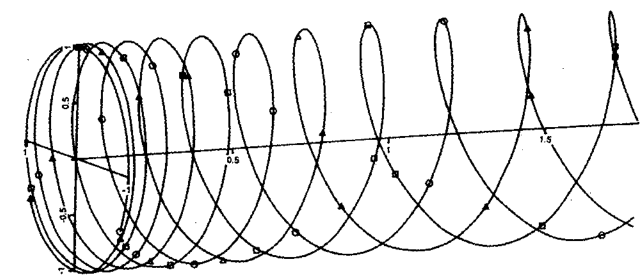
$$\frac{V}{U_o}=\frac{t/\tau_2}{1+(t/\tau_2)}, \qquad \frac{\omega}{\omega_o}=\frac{1}{1+(t/\tau_2)} \tag{10}$$

The solutions for this case, as represented by Eqs. (10), are the same as for the low swirl case with the exception of the time constant τ_2. The results, as expressed by Eqs. (10), are qualitatively the same as those shown in Fig. 1.

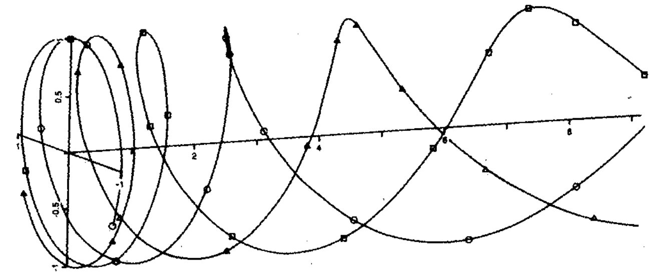
In order to illustrate the effect of the time constants on the bubble path, Figs. 2 show the effect of the time constant τ_o on the bubble path. Each of the figures shows the loci that the bubbles ejected at the edge of a three-bladed propeller in a low Reynolds number flow would acquire for four different values of the time constant τ_o. Note that, in order to include as much detail as possible, the axial distance x in the four figures is different. Similar results would be obtained if the bubble path were to be plotted for the high Reynolds number cases.

The low Reynolds number velocities have exponentially decaying form, whereas the high Reynolds number cases have an algebraically decaying relationship.

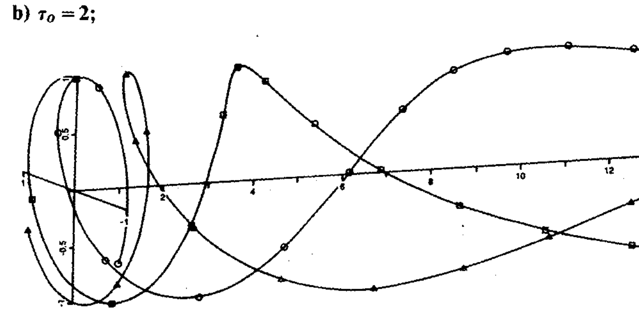
For the low Reynolds number case, the time constant has been determined to be roughly equivalent to the time required for a disturbance to propagate in a viscous fluid (i.e., Stokes' first problem). In the low swirl case the time constant found in the algebraic form of the solution has been found to be linearly related to the bubble size, whereas for the high swirl case, the time constant is inversely related to the initial angular velocity of the bubble and is independent of its size. The bubble immersed in a low Reynolds number environment has been shown to asymptote to its final conditions in a much quicker relative time and distance.



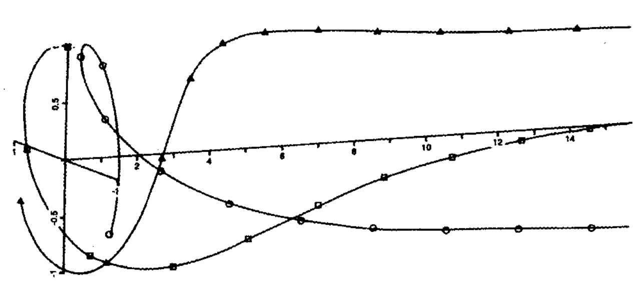
a) $\tau_o = 20$;



b) $\tau_o = 2$;



c) $\tau_o = 1$;



d) $\tau_o = 0.5$.

Fig. 2 Bubble path for low Reynolds numbers.

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[2]Newman, J. N., *Marine Hydrodynamics*, MIT Press, Cambridge, MA, 1977.

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